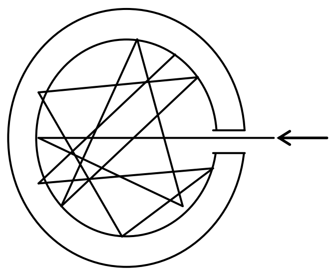
Determination of Stefan-Boltzmann Constant.

An object at some non-zero temperature radiates electromagnetic energy. For the perfect black body, which absorbs all light that strikes it, it radiates energy according to the black-body radiation formula.

The *black body* is an object that absorbs all electromagnetic radiation that falls onto its surface. The black body does not allow radiation passes through it, and the radiation can not be reflected, yet in classical physics, it can theoretically radiate any possible wavelength of energy. The black body emits light which is called *black-body radiation*. The amount and type of electromagnetic radiation emitted by a black body is connected to its temperature. Black bodies with temperatures below around 700 K (430 °C) produce a small amount of radiation at visible wavelengths, and because of this appear black. Above this temperature, the black bodies begin to produce radiation at visible wavelengths starting at red, after this going through orange, yellow, and white, and ending up at blue for an increased temperature.



Black body

In 1862 Gustav Kirchhoff introduced the term black body and three years ago gave his law of thermal radiation which is: *at thermal equilibrium, the emissivity of a body (or surface) equals its absorptivity (absorbance).*

The *emissivity* $\varepsilon_{\nu,T}$ of a material is the ratio of energy radiated by the material to energy radiated by a black body at the same temperature. Emissivity depends on temperature, emission angle, and wavelength. The *grey body assumption* states that surface's spectral emissivity and absorptivity do not depend on wavelength. The black body would have an $\varepsilon_{\nu,T} = 1$ while any real object would have $\varepsilon_{\nu,T} < 1$. The *absorptivity* (*absorbance*) $A_{\nu,T}$ is the fraction of incident light (power) that is absorbed by the body (surface). In more situations, emissivity and absorption may be defined to depend on wavelength and angle.

The mathematical statement of *Kirchhoff's law* is given by:

$$\varepsilon_{\nu,T} = A_{\nu,T}\,. \tag{1}$$

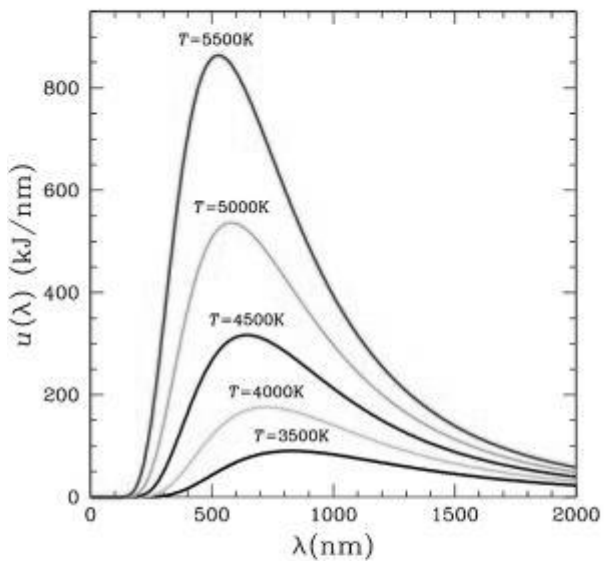
Other important laws are *Wien's displacement law*, *Wien's Radiation Law*, *Rayleigh-Jeans's law*, *Planck's law*, and the *Stefan-Boltzmann law* (*Stefan's law*).

Wien's displacement law is: *there is an inverse relationship between the wavelength of the peak of the emission of a black body and its temperature*

$$\lambda_{\max} \cdot T = b, \quad (2)$$

where $\lambda_{\max}$ is the peak wavelength in meters, T is the temperature of the blackbody in kelvin (K), and b is *Wien's displacement constant* which is $b = 2{,}8978 \cdot 10^{-3}$ m·K.

According to Wien's law an increasing in temperatures determines a shorter wavelength at which it will emit most of its radiation. Using Wien's law, the surface temperature of the Sun, which is 5778 K corresponds to a peak emission at a wavelength of 502 nm. This wavelength is in the middle of the visual spectrum of land animal, because of the spread resulting in white light.



Wien's law*

Wien's radiation law (1896) was based on classical physics and failed to fit the experimental data for short wavelengths, although fitting well for longer ones. It was later substituted by the Planck's law of black-body radiation, which worked for all wavelengths. *Wien's Radiation Law* is given by:

$$f(\lambda) = C_1 \lambda^{-5} e^{-C_2 / \lambda T}, \quad (3)$$

where C_1 and C_2 are constant.

The enunciation of *Stefan-Boltzmann law* (*Stefan's law*) is: the total energy radiated per unit surface area of a black body in unit time (the black-body *irradiance, energy flux density, radiant flux*, or the *emissive power*), j, is directly proportional to the fourth power of the black body's thermodynamic temperature *T* (absolute temperature)

$$j = \varepsilon_T \sigma T^4, \quad (4)$$

where ε_T is the *emissivity* of the black body which is $\varepsilon_T = 1$ for the perfect black body and $\sigma = 5{,}6697 \cdot 10^{-8}\ W / m^2 \cdot K^4$ is the *Stefan–Boltzmann constant* or *Stefan's constant*. The law was discovered experimentally by Jožef Stefan in 1879 and derived theoretically, using thermodynamics, by Ludwig Boltzmann in 1884**.** Using his law Stefan determined the temperature of the Sun's surface. Furthermore, the temperature of stars other than the Sun can be approximated using a similar means by treating the emitted energy as a black body radiation. Also, in a similar way can be calculated the temperature of the Earth.

For the black body the Stefan–Boltzmann law is given by:

$$j = \sigma T^4 . \tag{5}$$

This law is very useful for astronomers that can evaluate the radii of stars.

This is the law that is studied in laboratory using an electric balance (Wheatstone).

Rayleigh-Jeans's law expresses the energy density of blackbody radiation of wavelength λ as

$$f(\lambda) = 8\pi k \frac{T}{\lambda^4}, \tag{6}$$

where T is the temperature in kelvins and $k = 1{,}38 \cdot 10^{-23}\ J \cdot K$ is *Boltzmann's constant*.

The law is derived from classical physics arguments. Lord Rayleigh was the first who obtained the fourth-power dependence on wavelength in 1900. The inclusion of the proportionality constant, was due to Rayleigh and Sir James Jeans in 1905. Their law agrees with experimental measurements for long wavelengths. However it disagrees with experiment at short wavelengths, where it diverges and predicts an unphysical infinite energy density. This failure is known as the *ultraviolet catastrophe*. This means that an ideal black body at thermal equilibrium will emit radiation with infinite power. The term "ultraviolet catastrophe" was first used in 1911 by Paul Ehrenfest. The solution to this problem led to the development of an early form of quantum mechanics.

The problem was fixed in 1900 by Max Planck who obtained *Planck's law*, a different law which gives the energy density of blackbody radiation of wavelength λ:

$$f(\lambda) = \frac{8\pi hc}{\lambda^5} \frac{1}{\mathrm{e}^{\frac{hc}{\lambda kT}} - 1}, \tag{7}$$

where $f(\lambda)$ is the *spectral energy density*, $h = 6{,}626 \cdot 10^{-34}\ J \cdot s$ is *Planck's constant*, and $c = 3 \cdot 10^8\ m/s$. The law is expressed in terms of wavelength $\lambda = \frac{c}{\nu}$. The radiation field of a black body may be thought of as a photon gas. In this case the energy density would be one of the thermodynamic parameters of that gas.

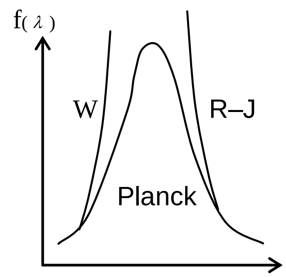
The Planck law does not suffer from an ultraviolet catastrophe, and agrees well with the experimental data. Planck's law contains as particular cases all the aforementioned laws. This law can be explained using the theory of *quanta* (*photons*) that states: *the emission is not a continuum process, is a quantified one*. Planck considered the possible ways of distributing electromagnetic energy over the different modes of charged oscillators in matter. He assumed that the energy of these oscillators consists of a set of discrete, integer multiples of a fundamental unit of energy, W, proportional to the oscillation frequency ν.

The energy of a quantum is given by:

$$W = W_i - W_f = h\nu , \tag{8}$$

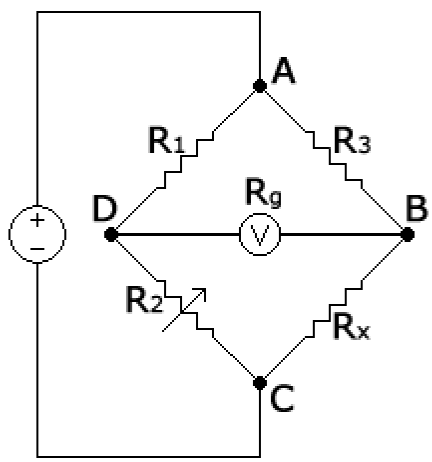
where $h\nu$ is the energy of a quantum, and W_i and W_f are the energies of oscillator before emission and after emission, respectively. Planck made this quantization assumption five years before Albert Einstein hypothesized the existence of photons (explaining the photoelectric effect). Planck assumed that the quantization can be applied only to the tiny oscillators that were thought to exist in a small limited region (atoms). He did not make assumption that light itself propagates in packets of energy (photons).

In the figure below there are presented the graphs for Wien's displacement law, Rayleigh-Jeans's law and the Planck's law.



Apparatus and Equipment

Electric balance (Wheatstone bridge), in the laboratory the measurements are made for values of voltage greater than $U = 40\,V$, using a bulb with wolfram filament.



Wheatstone bridge

Measurements

1. For the electric balance (Wheatstone bridge) one gets:

$$RI^2 = eS\sigma T^4 \qquad (9)$$

2. Picking-up the values of resistance R_3 which equilibrates the electric balance (Wheatstone bridge) for values of voltage greater than $U = 40\,V$.

3. Calculations of $R = \frac{R_1}{R_2} R_3$, current intensity $I = \frac{U}{R}$, and absolute temperature T.

4. Determining a and b of the straight line $\ln(RI^2) = \ln(eS\sigma) + 4\ln T$. Making the graph with $\ln T$ on x-axis and $\ln(RI^2)$ on y-axis, and calculating the points using a and b.

5. Evaluating the Stefan–Boltzmann constant from b.

Results

No. crt.	$U\ (V)$	$R_3\ (\Omega)$	$R\ (\Omega)$	$I\ (A)$	$T\ (K)$	$\ln T$	$\ln(RI^2)$